Enhancement of Signal to Noise Ratio Using Bispectrum A Quantitative Analysis for Very Low SNR

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Abstract- **Bispectrum has been widely used to enhance the SNR. This is based on the assumption that the HOS properties of the signal of interest are different to the HOS properties of the noise. In the present work, we consider the use of Bispectrum techniques when repeated measurements are made of a deterministic signal embedded in random noise where SNR is in the range from -17dB up to 0dB. The performance of estimators and reconstruction algorithms are evaluated using simulated evoked potential data. We conclude that the best performance is achieved if we reconstruct the phase by bispectrum and amplitude through a second order method such as 'Spectral Subtraction'.**
Keywords- Bispectrum Reconstruction, SNR Enhancement, ABR

I. INTRODUCTION

Many authors have investigated the technique of bispectrum averaging to improve the signal-to-noise ratio of signals that are heavily corrupted with noise [1,2,3]. The method is based on recovering deterministic signals from the averaged bispectrum of noisy observations of the signal.

These researchers have considered cases where SNR is better than 0dB. This paper emphasizes on the situations where the SNR is in the range from -17dB up to 0dB. This value of the SNR is very typical in the recording of Auditory Brainstem Response (ABR) in clinical applications.

In this communication, it is assumed that a large number N of a deterministic signal s(t) are observed with a random delay in a noisy environment. In our application the deterministic signal will be ABR.

Each realization $x_i(t)$ is modeled using the following expression:

$$x_i(t) = s_i(t) + n_i(t)\ ,\ 0 \le t \le N-1 \tag{1}$$

where $n_i(t)$ is the background noise assumed to be guassian and N is the number of data points per response. We simplified the ABR model introduced in [4] and simulate the ABR as follows:

$$s_i(t) = \sum_{j=1}^{P} [a_j\, \rho(t-\tau_i) + b_i^j(\tau_i)] \tag{2}$$

where $\rho(t)$ represents the basic peak component of the ABR waveform, P stands for the number of peak components involved in the ABR waveform, τ_i and $b_i(\tau_i^j)$ are random variable for the latency and vertical shift of the jth component.

In the simulation, N = 256 (A typical number in clinical recording of ABR), P = 5, a_j = [0.7 0.5 0.8 1 0.9], τ_i a random delay with gaussian probability density function of mean zero and standard deviation of $\pm\sqrt{0.4}$ ms and $b_i^j(\tau_i)$ is a gaussian delay of zero mean and standard deviation of 0.075.

A Fourier filter same as [4] is used to smooth the simulated signal. The following autoregressive model simulates the background noise:

$$n_i(t) = 1.508\,n_i(t-1) - 0.1587\,n_i(t-2) - 0.3109\,n_i(t-3) - 0.051\,n_i(t-4) + \omega(t) \tag{3}$$

Using the samples generated by the model, in the following sections we will evaluate the performance of different bispectrum estimators and then compare the reconstruction methods for recovery of Fourier phase and amplitude from bispectrum.

II. METHOD

A. Evaluation of the Estimators

For the evaluation of different Bispectrum estimators, the performance on suppression of additive guassian noise is compared.

We estimate the Bispectrum of $x_i(t)$ by three estimators: Direct estimator, Unbiased Indirect estimator and Biased Indirect estimator. For the analysis, we estimate the Bispectrum of $x_i(t)$ and average the Bispectrum. If there are R realizations we then have [5]:

$$\frac{1}{R}\sum_{r=1}^{R} \hat{B}_x^{(r)}(k,l) = B_s(k,l) + B_n(k,l) \tag{5}$$

Because the noise is gaussian, in ideal situation $B_n(k,l)$ is zero. Due to finite data length used to estimate the Bispectrum $\frac{1}{R}\sum_{r=1}^{R} \hat{B}_x^{(r)}(k,l)$ is not equal to true value $B_x(k,l)$ and there is always an estimation error $\Delta_x(k,l)$:

$$\frac{1}{R}\sum_{r=1}^{R} \hat{B}_x^{(r)}(k,l) = B_x(k,l) + \Delta_x(k,l) \tag{6}$$

To show the influence of suppression of additive gaussian noise the ratio of error $\Delta_x(k,l)$ to true value $B_x(k,l)$ in different SNR is calculated.

In ideal situation $B_x(k,l)$ shall be equal to $B_s(k,l)$ so by using (6) we can estimate the error as follows:

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$$\Delta_x(k,l) = \frac{1}{R}\sum_{r=1}^{R} \hat{B}_x^{(r)}(k,l) - B_s(k,l) \qquad (7)$$

The test ratio is defined as:

$$R = \frac{\sum_k \sum_l |\Delta_x(k,l)|}{\sum_k \sum_l |B_x(k,l)|} = \frac{\sum_k \sum_l \left| \frac{1}{R}\sum_{r=1}^{R} \hat{B}_x^{(r)}(k,l) - B_s(k,l) \right|}{\sum_k \sum_l |B_s(k,l)|} \qquad (8)$$

B. Evaluation of the Reconstruction Methods

The Bispectrum reconstruction methods have been introduced and investigated by many authors, but again they have been assessed for SNRs better than 0dB. As part of this work the performance of Bispectrum reconstruction methods have been evaluated for low SNRs. We analyzed the Fourier phase reconstruction and Fourier amplitude reconstruction from the bispectrum separately in order to find out which one is more prone to noise. The correlation coefficient as defined in [6] was selected as performance criteria for the reconstruction method.

III. RESULTS

Using the samples generated by the models defined by (2) and (3), the ratio R was calculated for different estimators, in SNR from –15dB to 3dB. The test was done 10 times and the ratios were averaged. Figure 1 shows the result. As can be seen the Biased Indirect Estimator has the lowest error.

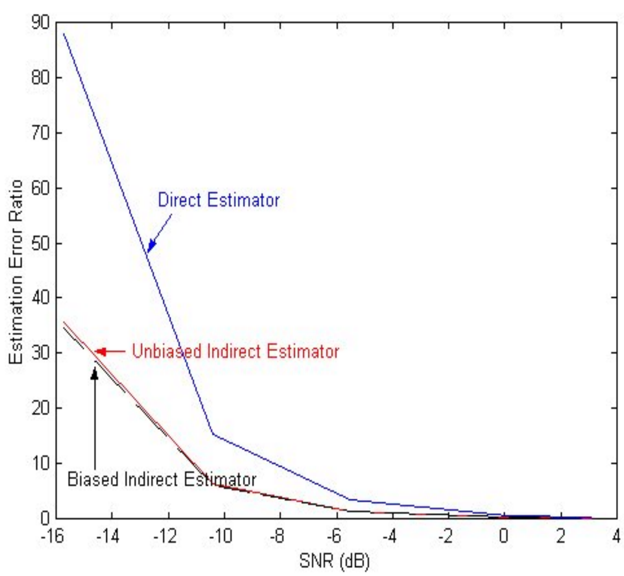

Figure 1- Averaged Normalized Estimation Error.

In order to evaluate the reconstruction methods, two phase-reconstruction and two amplitude reconstruction methods were evaluated on a 300 simulated data set at SNRs between –16dB up to 0dB.

The Mastsuoka-Ulrich [1] and a recursive method that uses information of averaged signal [2] were implemented on the same set of data. In order to assess the performance of phase reconstruction algorithms independently, the true amplitude of simulated ABR was used. Figure 2 and 3 show the reconstructed signals using the abovementioned methods.

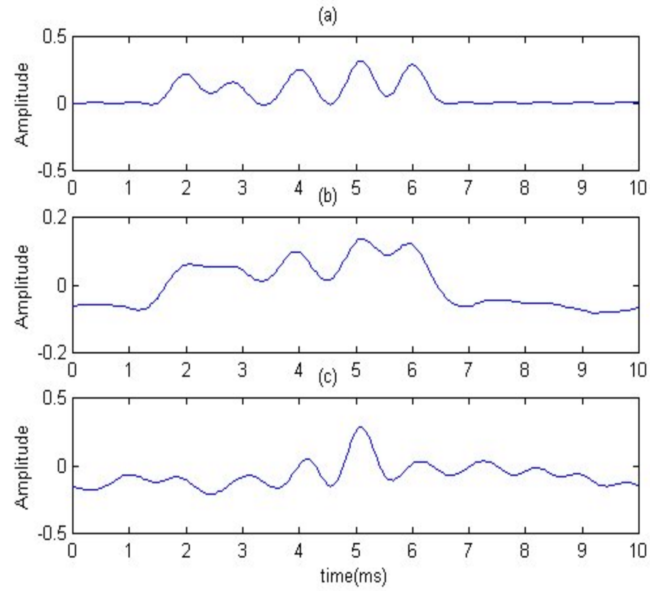

Figure 2- Evaluation of phase reconstruction by MU method. (a) True ABR, Reconstructed signal by (b) simple averaging (c) MU method

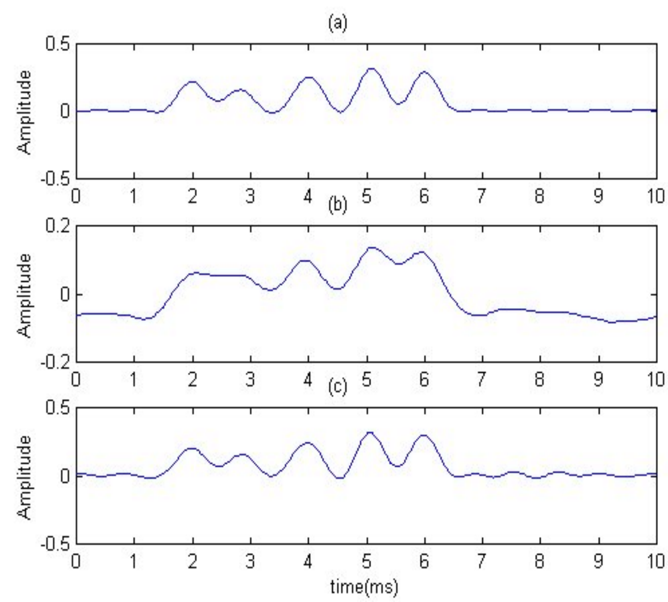

Figure 3- Evaluation of phase reconstruction by recursive method using averaged signal information [2]. (a) True ABR, Reconstructed signal by (b) simple averaging (c) recursive method.

The much better performance of the recursive method is obvious. Figure 4 shows the reconstructed signal using recursive phase and true ABR amplitude at different SNR levels. As seen the SNRs has minimum effect on this phase reconstruction method.

Same as phase reconstruction, two amplitude reconstruction methods, the closed-form approach [3] and the Least-Square [3] approach were simulated and the true phase was used to reconstruct the signals. Due to low SNR both algorithms could not provide the desired responses and failed. Figure 5 shows the reconstructed signal by LS approach at SNR = -12 dB. The result obtained is not satisfactory.

To reconstruct the amplitude, we considered the use of a second order method rather than higher order [7]. This method shall (1) increase the SNR and (2) be insensitive to linear phase shift (which is common in sequential ABR measurements). The ‘Spectral Subtraction’ method was selected to reconstruct the amplitude. This method requires estimation of the background noise spectrum. Recording the pre stimulus data to estimate the noise spectrum in clinical application can do this. It is obvious that the phase information is lost by this method but we reconstruct the phase using Bispectrum.

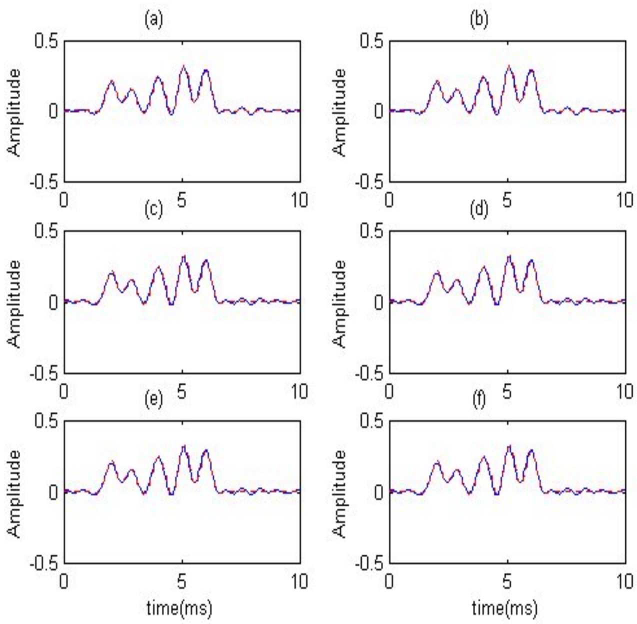

Figure 4- Evaluation of phase reconstruction by recursive methods at different SNRs. (a) –16 dB (b) –15 db (c)-13 dB (d) –7 dB (e) –2 dB (f) 0 dB

Figure 6 shows the reconstructed signal by combining the ‘Spectral subtraction’ method for amplitude recovery and Recursive phase. The ABR peak locations and amplitudes are much better revealed by these methods than the averaging.

To compare the performance of this method to simple averaging, the following test was executed:

10 separate data sets of 300 signals (ABR + Noise) were generated for each different SNRs between -18dB up to 0dB. Signals were reconstructed using the proposed method and simple averaging. The correlation coefficient between the reconstructed signals and the true ABR were calculated and averaged for each data set. Figure 7 shows the averaged correlation coefficient against different SNRs. The result shows the proposed method outperform the averaging.

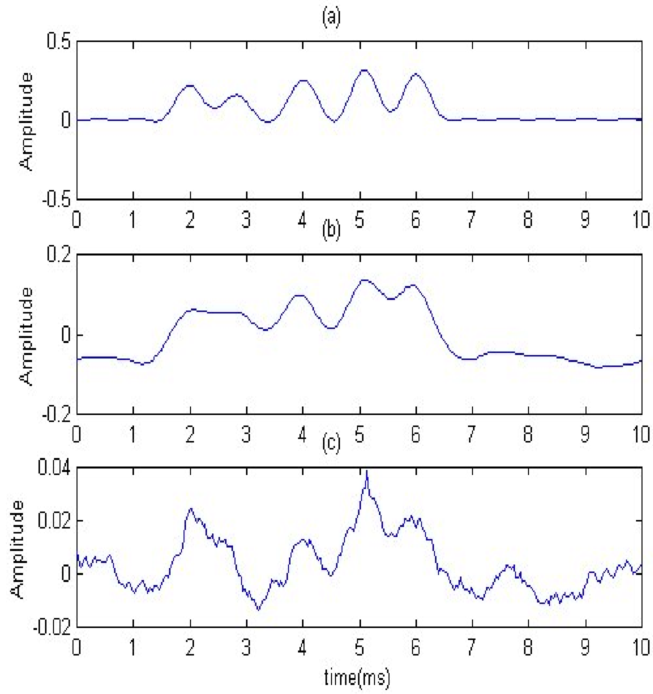

Figure 5- Evaluation of amplitude reconstruction by LS approach. (a) True ABR (b) simple averaging (c) LS approach

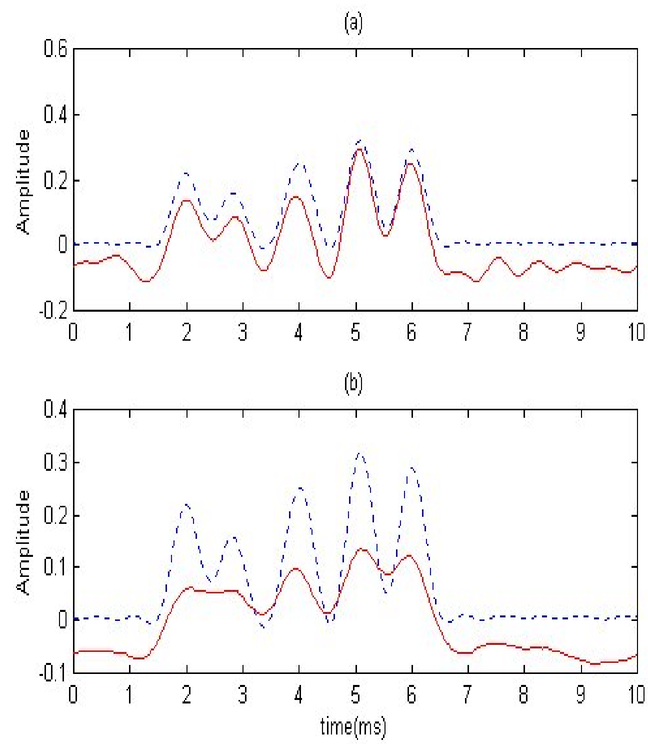

Figure 6- Comparison of True ABR (dotted line) to (a) reconstructed signal using ‘Spectral Subtraction’ and Recursive phase using bispectrum and (b) simple averaging

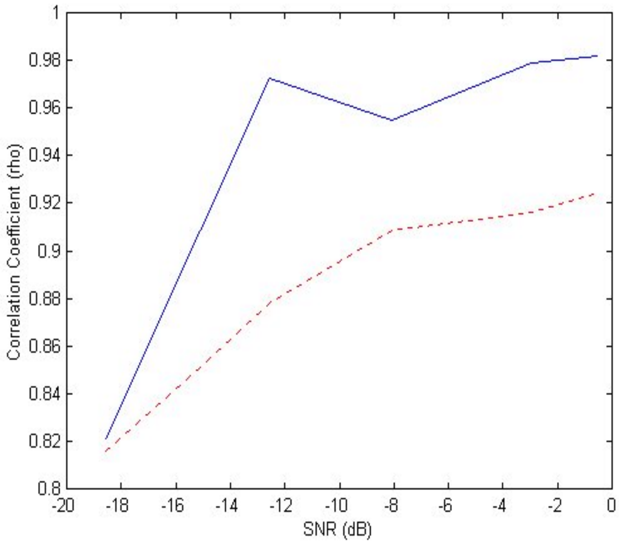

Figure 7- Averaged correlation coefficient against different SNRs for proposed method (solid line) and the averaging (dotted line)

IV. CONCLUSION

The performance of bispectrum estimators as well as the Fourier amplitude and phase reconstruction methods from bispectrum deviates by decreasing the SNR. We evaluated these performances at different SNRs. The result shows that the recursive phase reconstruction algorithm using the average signal information does not deviate much by the decrease in SNR, however the amplitude reconstruction methods all fail in low SNR. To overcome this, use of second order algorithm such as 'Spectral Subtraction' has been proposed. The performance of the proposed combination has been assessed and showed that this method outperforms the averaging.

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